HEAT TRANSFER IN THE NONISOTHERMAL FLOW OF AN ANOMALOUSLY VISCOUS FLUID IN A HELICAL DUCT

A. I. Mumladze, Yu. G. Nazmeev, and O. V. Maminov

UDC 532.517.2

The problem of heat transfer in the initial section of a helical duct with a steady flow of an anomalously viscous fluid is solved numerically.

A standard technique for the intensification of convective heat transfer in anomalously viscous media is to impart a swirling motion of the flow by means of helicoid inserts. We do not know of any theoretical studies of heat transfer in helical flows of non-Newtonian media.

In this article we solve the heat-transfer problem in the initial section of a helical duct formed by an internal continuous helicoid vane fitted tightly to the inside wall of the pipe (Fig. 1) for steady laminar flow of an anomalously viscous temperature-dependent fluid under the condition of dissipative heat loss and Dirichlet-type boundary conditions.

Assuming that the heat transfer along the axis of the duct by conduction is immeasurably small in comparison with the forced transfer, we can state the problem as follows:

$$\frac{v_\varphi(r,\ \varphi,\ u)}{r}\ \frac{\partial u}{\partial \varphi} + v_z(r,\ \varphi,\ u)\frac{\partial u}{\partial z} = a\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\ \frac{\partial u}{\partial r} + \frac{1}{r^2}\ \frac{\partial^2 u}{\partial \varphi^2}\right) + \frac{E(r,\ \varphi,\ u)}{\rho C_p},\ r,\ \varphi \in \bar{G},\ 0 \leqslant z \leqslant Z \quad (1)$$

with the boundary conditions

$$u(r,\ \varphi,\ 0) = u_0,\ u(r,\ \varphi,\ z)|_\Gamma = u_\Gamma. \quad (2)$$

To determine the profiles of the axial and circumferential components of the velocity we write the system of equations of motion and continuity, assuming that the radial velocity component v_r is zero, the circumferential component has the form $v_\varphi = \omega(r,\ \varphi)r$, the tangential pressure gradient is insignificant, and the force of gravity is small in comparison with the centrifugal force:

$$\frac{\partial p}{\partial r} = \omega^2 r, \quad (3)$$

$$\frac{\partial p}{\partial z} + \omega\frac{\partial v_z}{\partial \varphi} = \frac{\partial}{\partial r}\left(\mu\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\ \frac{\partial}{\partial \varphi}\left(\mu\ \frac{\partial v_z}{\partial \varphi}\right) + \frac{\mu}{r}\ \frac{\partial v_z}{\partial r} \quad (4)$$

with the boundary conditions

$$v_z(r,\ \varphi)|_\Gamma = 0,\ v_\varphi(r,\ \varphi)|_\Gamma = 0. \quad (5)$$

We seek a solution of the system (3)-(5) in a cylindrical coordinate system rotating with an angular velocity ω about the z axis and moving translationally along the x axis, which coincides with the axes of the pipe and the internal vane.

Inasmuch as the angular velocity of rotation of the coordinate system depends on the fluid flow velocity and on the pitch of the helicoid vane, it can be written in the form

$$\omega(r,\ \varphi) = v_z/S. \quad (6)$$

The axial component of the velocity for the given coordinate system is $v_z = v_{z_{syst}} + v_{z_{rel}}$, where the velocity of the system can be expressed in the form $v_{z_{syst}} = v_{z_{rel\ max}}$.

S. M. Kirov Chemical-Technology Institute, Kazan. Translated from Inzhenerno Fizicheskii Zhurnal, Vol. 43, No. 2, pp. 228-234, August, 1982. Original article submitted May 27, 1981.

0022-0841/82/4302-0869$07.50

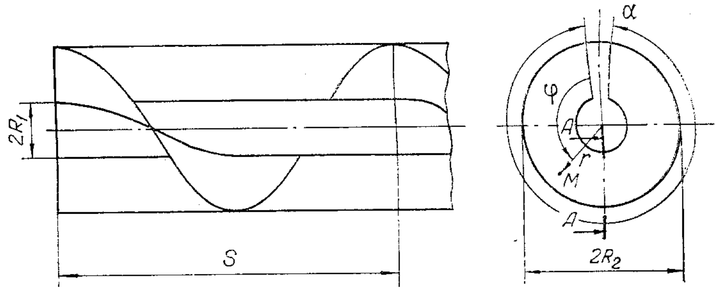

Fig. 1. Schematic view of the helical duct.

The axial velocity component as the function $v_{z_{rel}} = f(r, \varphi)$ is determined from the expression

$$dv_{z_{rel}} = \frac{\partial v_{z_{rel}}}{\partial r} dr + \frac{\partial v_{z_{rel}}}{\partial \varphi} d\varphi. \tag{7}$$

Assuming that the components of the tangential stress are equal to

$$\tau_r = \mu \frac{\partial v_{z_{rel}}}{\partial r} = \frac{\Delta p}{2} \frac{\partial \psi}{\partial r}, \quad \tau_\varphi = \mu \frac{\partial v_{z_{rel}}}{\partial \varphi} = \frac{\Delta p}{2} \frac{\partial \psi}{\partial \varphi} \tag{8}$$

and then substituting this expression into the total differential (7), we represent the distribution of the axial component of the velocity over the cross section of the helical duct by a curvilinear integral:

$$v_{z_{rel}}(r, \varphi) = \frac{\Delta p}{2} \int \Phi \left(\frac{\partial \psi}{\partial r} dr + \frac{\partial \psi}{\partial \varphi} \partial \varphi \right), \tag{9}$$

where the fluidity Φ is expressed by the relation

$$\Phi = f(\tau^2) = f(\tau_r^2 + \tau_\varphi^2) = f\left\{ (0.5\Delta p)^2 \left[\left(\frac{\partial \psi}{\partial r} \right)^2 + \left(\frac{\partial \psi}{\partial \varphi} \right)^2 \right] \right\}. \tag{10}$$

For the specific rheological law we use the relation between the fluidity and the shear stress:

$$\Phi = \Phi_0 (1 + k\tau^m). \tag{11}$$

Expression (9) describes the distribution of the axial velocity component in the relative coordinate system for a duct whose cross-sectional configuration corresponds to a known solution of the Dirichlet problem for ψ in the Poisson equation [1]. For a duct with the given cross-sectional configuration the function ψ is written [2]

$$\psi = \frac{32}{\pi^4} \sum \sum \frac{1}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) mn} \sin \frac{m\pi\varphi}{a} \sin \frac{n\pi r}{b}, \tag{12}$$

where $a = \frac{\pi (R_1 + R_2)}{360} \alpha$, $b = R_2 - R_1$.

Having determined the velocity and stress components, we can write the mechanical energy dissipation function for the given problem:

$$E(r, \varphi) = \tau_r \frac{\partial v_z}{\partial r} + \tau_\varphi \frac{\partial v_z}{\partial \varphi}. \tag{13}$$

To close the system of analytical equations we use the Arrhenius formula for the viscosity as a function of the temperature [3]:

$$\Phi_0 = A \exp\left(-\frac{B}{RT} \right). \tag{14}$$

To solve Eq. (1) subject to the boundary conditions (2) we use the same notation as in [4]:

$$v_{\chi_1}(\chi, u)\frac{\partial u}{\partial \chi_1} + v_z(\chi, u)\frac{\partial u}{\partial z} = aLu + \frac{E(\chi, u)}{\rho C_p G}, \tag{15}$$

$$\chi = (\chi_1, \chi_2),\ L = \sum_{\beta=1}^{2} L_\beta,\ L_\beta u = \frac{\partial^2 u}{\partial \chi_\beta^2},\ \chi \in \bar{G},\ 0 \leqslant z \leqslant Z,$$

$$u(\chi, z)|_\Gamma = u_\Gamma,\ u(\chi, 0) = u_0. \tag{16}$$

To facilitate the numerical solution we make a transformation of coordinates, shifting the origin so that the domain $\bar{G}$ will be situated in the first quadrant of the Cartesian coordinate system.

The following conditions are satisfied in the investigated domain $\bar{G}$: 1) The intersection of $\bar{G}$ with any straight line parallel to one of the coordinate axes consists of a finite number of intervals; 2) it is possible to construct in $\bar{G}$ a connected net $\bar{\omega}_h$ with spacing h_β, $\beta = 1, 2$.

The set ω_h of interior nodes of the net consists of the points $\chi=(\chi_1, \chi_2)\in G$ of intersection of the lines $\chi_\beta = i_\beta h_\beta$, $i_\beta = 0, \pm 1, \pm 2, \ldots$, $\beta = 1, 2$, and the set γ_h of boundary nodes consists of the points of intersection of the lines C_β, $\beta = 1, 2$, passing through all interior nodes $\chi \in \omega_h$ with the boundary Γ. For the difference approximation of the operator L_β at the node χ we use a three-point stencil consisting of the points $\chi^{(-1\beta)}$, χ, $\chi^{(+1\beta)}$.

The difference operator $\Lambda_\beta \sim L_\beta$ has the form

$$\frac{\partial u}{\partial \chi_1} \sim \Lambda_1 y = \frac{y^{(+1_\beta)} - y^{(-1_\beta)}}{h_{1+}^* + h_{1-}^*}, \tag{17}$$

$$\frac{\partial^2 u}{\partial \chi_\beta^2} \sim \Lambda_\beta y = \frac{1}{h_\beta^+}\left(\frac{y^{(+1_\beta)} - y}{h_{\beta+}^*} - \frac{y - y^{(-1_\beta)}}{h_{\beta-}^*}\right),\ h_\beta^+ = 0.5\,(h_{\beta+}^* + h_{\beta-}^*). \tag{18}$$

In the interval $0 \leqslant z \leqslant Z$ we introduce the net $\bar{\omega}_{\tau^*} = \{z_j = j\tau^*,\ j = 0, 1, \ldots, j_0\}$ with spacing $\tau^* = z/j_0$. In the layer (z^j, z^{j+1}) we solve in succession the equations

$$v_z \frac{\partial v_{(1)}}{\partial z} + v_{\chi_1}\frac{\partial v_{(1)}}{\partial \chi_1} = a\frac{\partial^2 v_{(1)}}{\partial \chi_1^2} + F \tag{19}$$

with the boundary condition

$$v_{(1)}(\chi, u^j) = v_{(2)}(\chi_2, u^j) \tag{20}$$

and the equation

$$v_z \frac{\partial v_{(2)}}{\partial z} = a\frac{\partial^2 v_{(2)}}{\partial \chi_2^2} + F$$

with the boundary condition $v_{(2)}(\chi, u^{j+1/2}) = v_{(1)}(\chi, u^{j+1/2})$. We set

$$v_{(2)}(\chi, 0) = u_0,\ F = \frac{E}{2\rho C_p G},\ v_z(\chi, u^j) = v_z(\chi, u^{j-1}),\ v_z(\chi, u^0) = v_z(\chi, u_0), \tag{21}$$

$$v_{\chi_1}(\chi, u^j) = v_{\chi_1}(\chi, u^{j-1}),\ v_{\chi_1}(\chi, u^0) = v_{\chi_1}(\chi, u_0). \tag{22}$$

We replace (19) and (20) by the double-layer purely implicit scheme

$$\bar{v}_z \frac{y^{j+1/2} - y^j}{\tau^*} + \bar{v}_{\chi_1}\Lambda_1 y^{j+1/2} = \bar{a}\Lambda_1 y^{j+1/2} + \bar{F}, \tag{23}$$

$$\bar{v}_z \frac{y^{j+1} - y^{j+1/2}}{\tau^*} = \bar{a}\Lambda_2 y^{j+1} + \bar{F}. \tag{24}$$

We augment (23) and (24) with the boundary conditions

$$y^{j+\beta/2} = u_\Gamma,\ \chi \in \gamma_{h_\beta},\ y(\chi, 0) = u_0. \tag{25}$$

For $y^{j+\beta/2}$ we obtain the boundary-value problem

$$A_{i_\beta} y_{i_\beta - 1}^{j+\beta/2} - C_{i_\beta} y_{i_\beta}^{j+\beta/2} + B_{i_\beta} y_{i_\beta + 1}^{j+\beta/2} = -F_{i_\beta}^{j+\beta/2},\ y^{j+\beta/2} = u_\Gamma \text{ for } \chi \in \gamma_{h_\beta},\ \beta = 1, 2, \tag{26}$$

which is solved by a double-sweep procedure and in which only the variable subscripts are indicated.

To solve the problem we have written a pL/1 program for Unified Series ("ES") computers [unified system used in COMECON nations]. The set ω_h is represented by a p × n rectangular matrix, where $p = \max(i_1 + 1)$, $n = \max(i_2 + 1)$.

Introducing the set $H_\beta = \{h^*_{i\beta}, N'_{i\beta}, N''_{i\beta}\}$, $i_\beta = 1, 2, 3, \ldots$, $\beta = 1, 2$, we can specify the initial and final values of the subscripts of the sweep coefficients in the difference equation (26).

For the solution of the problem we form the matrices of like orders p, n, M_1, M_2, M_3, Dux, Duy, v_z, v_{χ_1}, Φ, whose elements are, respectively, the values of the functions y^j, $y^{j+1/2}$, y^{j+1}, $\partial\psi/\partial\chi_1$, $\partial\psi/\partial\chi_2$, v_z, v_χ; Φ.

At the start of the calculations we assign the elements of M_1 the values $y(\chi, 0) = u_0$. In calculating the elements of M_2 the sweep process is in the direction of χ_1, and for M_3 it is in the direction of χ_2, in which case the set H_β is used. If the difference between the values of the elements of the matrices M_1 and M_3 is greater than a specified ε, the elements of M_1 are assigned the values of the elements of M_3, and the elements of M_2 are computed, etc. If this difference is smaller than the specified ε, the computation is terminated.

As an example we calculate the velocity and temperature field for the flow of a model anomalously viscous fluid (8.5% aqueous solution of sodium carboxymethyl cellulose Na-CMC) with the following parameters: k = 9.165; m = 0.681; A = 0.9; B = 35 kJ/mole.

The rheological parameters of the model fluid are determined from the results of viscometric measurements performed as part of an experimental study of heat transfer in helical ducts. The calculations are carried out for a duct with $R_1 = 0.021$ m, $R_2 = 0.036$ m, S = 0.080 m.

Figure 2 shows the dimensionless profiles of the axial and circumferential velocity components in the annular cross section at $r = (R_1 + R_2)/2$ and in the radial cross section A—A. Inasmuch as the calculations are carried out with regard for the temperature dependence of the viscosity and dissipative heat release, the velocity profiles are deformed along the length of the duct and are shown in Fig. 2 for several values of $Z = z/l$, where l is the length of the duct.

Two flow regimes are possible in the nonisothermal case: constant volumetric flow rate (Q = const) and constant axial pressure gradient (Δp = const). We are investigating the second regime, which in the practical situation of a fluid with temperature-dependent parameters is realized for a constant mass flow of fluid. The average flow velocity needed for the graphs is determined in each investigated z-cross section in terms of the volumetric flow rate through the cross section

$$Q = \frac{1}{\Delta p}\iint \Phi\cdot\tau^2 dr d\varphi \tag{27}$$

according to cubature formulas, within prescribed error limits, by means of the useful cross section

$$F = S\left[\sqrt{R_2^2 + \left(\frac{S}{2\pi}\right)^2} - \sqrt{R_1^2 + \left(\frac{S}{2\pi}\right)^2}\right], \tag{28}$$

which is defined as in [5].

Figure 3 shows the variation of the dimensionless temperature $\theta = (u - u_\Gamma)/(u_0 - u_\Gamma)$ in the same cross sections of the helical duct for the same values of z.

Figure 4 shows the dynamics of the variation of the local heat-transfer coefficient α^* along the length of the duct.

A comparison of the calculated and experimental values (according to the results of tests with 8.5% Na-CMC) of the average heat-transfer coefficients shows that the maximum discrepancy between them is 24% and is attributable to the accuracy of description of the model-fluid flow curves by the rheological equations (11) and (14), the assumption of m =

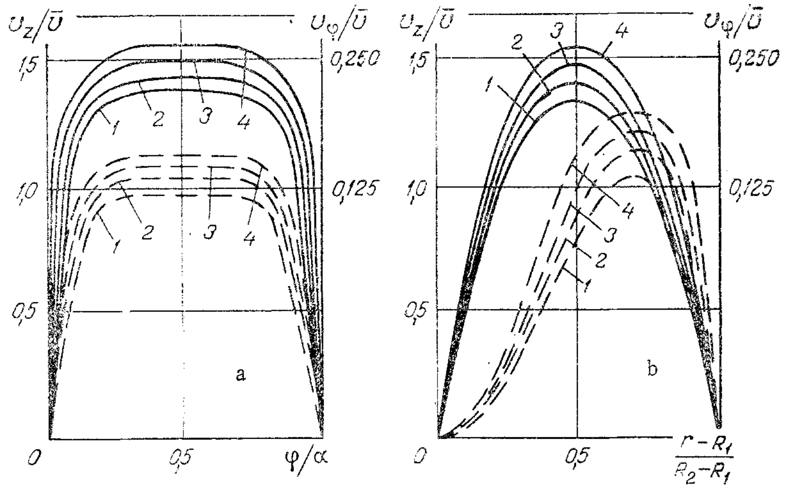

Fig. 2. Calculated profiles of the axial component v_z (solid curves) and circumferential component v_φ (dashed curves) in a flow of 2.5% Na-CMC. a) In the annular cross section at $r = (R_1 + R_2)/2$; b) in the radical cross section A–A; 1) z = 0.0016; 2) 0.32; 3) 0.64; 4) 1.0; Δp = 100 N/m³.

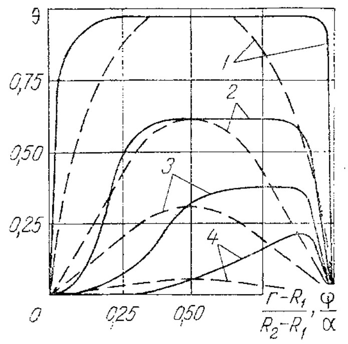

Fig. 3

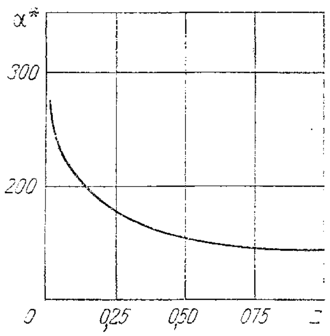

Fig. 4

Fig. 3. Calculated profiles of the dimensionless temperature θ in an annular cross section (solid curves) and in the radial cross section A–A (dashed curves). (Same nomenclature as Fig. 2.)

Fig. 4. Dynamics of the variation of the local heat-transfer coefficient α^*, W/m²K, along the length of the helical duct (Δp = 100 N/m³).

n = 5 in (12), and the assumption that a Dirichlet-type boundary conditions is proper on the surface of the edge of the helicoid vane.

NOTATION

r, φ, z, running coordinates; u, instantaneous temperature; u_0, initial temperature of the fluid; u_Γ, duct wall temperature; v_z, v_φ, axial and circumferential components of the flow velocity; a, ρ, C_p, thermal diffusivity, density, and heat capacity of the fluid; E, mechanical energy dissipation function; Γ, contour of the duct; ω, angular velocity of rotation; p, pressure; μ, effective viscosity of the fluid; S, pitch of the internal helicoid vane; $v_{z_{syst}}$, axial velocity of the coordinate system; $v_{z_{rel}}$, axial component of the velocity in a moving coordinate system; τ_r, τ_φ, components of the shear stresses; Δp, pressure drop per unit length of the duct; ψ, function characterizing the duct geometry and satisfying boundary conditions analogous with $c\bar{v}(r, \varphi)$ (representing the solution of the Dirichlet problem in the Poisson equation); Φ, fluidity; Φ_0, fluidity as $\tau \to 0$; τ, shear stress; k, m, constants in the rheological equation (11); R_1, radius of the outer surface of the central tube of the helicoid vane; R_2, radius of the inner surface of the outer tube; α, central angle; A, a constant; B, activation energy for the flow process; R, gas constant; T, absolute temperature; $\chi = (\chi_1, \chi_2)$, point in two-dimensional Euclidean space; h_β, spacing of computing net ω_h; ω_h, set of interior nodes; γ_h, set of boundary nodes; G, a domain; $\overline{G}$, a domain with boundary Γ; $L_\beta u$, Laplace operator; C_β, straight line through an interior node; Λ, difference operator; τ^*, z-spacing of net; h_β^*, distance from irregular node χ to boundary node $\chi^{(-1\beta)}$ or $\chi^{(+1\beta)}$; $h_{i\beta}^*$, distance from nodes $\omega_{h,\beta}$ next to the boundary to boundary nodes $h_{h,\beta}$; $N'_{i\beta}$, indices of left boundary nodes in a matrix in the direction of χ_β; $N''_{i\beta}$, indices of right boundary nodes; $A_{i\beta}$, $B_{i\beta}$, $C_{i\beta}$, sweep coefficients in difference equation; Q, volumetric flow rate; θ, dimensionless temperature; $\bar{v}$, average flow velocity; Z, duct length; l, length of the initial thermal section; F, useful cross section of duct; $\bar{v}_{\chi_1}$, $\bar{v}_z$, $\bar{\alpha}$, $\bar{F}$, discrete values of functions.

LITERATURE CITED

1. K. D. Vachagin and Yu. G. Nazmeev, "Calculation of the velocity profile for flow of an anomalously viscous fluid in cylindrical ducts of arbitrary cross section," Inzh.-Fiz. Zh., 34, No. 6, 1081-1084 (1978).
2. A. N. Dinnik, Torsion [in Russian], NKTP SSSR, Moscow—Leningrad (1938).
3. G. V. Vinogradov and A. Ya. Malkin, Rheology of Polymers [in Russian], Khimiya, Moscow (1977).
4. A. A. Samarskii, Introduction to the Theory of Difference Schemes [in Russian], Nauka, Moscow (1971).
5. Yu. G. Nazmeev and N. A. Nikolaev, "Generalization of experimental data on heat transfer in pipes fitted with ribbon swirling vanes," Teploenergetika, No. 3, 51-53 (1980).

TRANSIENT PROCESSES IN SHEAR FLOWS OF A VISCOELASTIC FLUID.

III. ELASTIC RECOVERY*

Z. P. Shul'man, S. M. Aleinikov, and B. M. Khusid

UDC 532.135

A mathematical description of the elastic recovery effect associated with viscoelastic shear flow is given.

In this article we investigate the transient process associated with the shear flow of a viscoelastic fluid in the clearance space between coaxial cylinders (problem 3) [1] when the outer cylinder is rigidly fixed and the inner one set in motion by the action of a constant external torque. The applied torque acts for a period of time t*, after which it is removed. The detailed mathematical statement of the problem and a procedure for its numerical solution are described in [1].

The external torque drives the cylinder and the fluid. As in problem 2, after several transits of a shear wave across the clearance space a quasisteady fluid flow regime is established, for which the conditions of realization are described in [2]. We now give a qualitative analysis of the influence of the rheological properties of the fluid on the laws

*The problem treated in Parts I and II [1, 2] and the present article are discussed in application to the Bird—Carreau, Meister, and MacDonald—Bird—Carreau nonlinear models in a paper by Z. P. Shul'man, S. M. Aleinikov, and B. M. Khusid, Rheodynamics and Heat Transfer in Unsteady Shear Flows of Nonlinear Hereditary Media, Preprint No. 6 of the Institute of Heat and Mass Transfer of the Academy of Sciences of the Belorussian SSR [in Russian], ITMO AN BSSR, Minsk (1982).

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Belorussian Polytechnic Institute, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 2, pp. 234-238, August, 1982. Original article submitted May 8, 1981.

 0022-0841/82/4302-0874$07.50